On the Gravitation of Exotic Matter

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Exotic matter is widely believed to produce repelling gravitational fields simply because the gravitational mass carries a minus sign. In a previous paper, however, it was demonstrated that although exotic matter has negative mass it resists acceleration, just like ordinary matter does when acted upon by external forces. This calls into question whether or not the gravitational properties of exotic matter are any different from the properties of ordinary matter. Herein, Newtonian gravitation is used to determine whether or not exotic matter exhibits repelling gravitational phenomena. It is found that the gravitational mass of exotic matter differs from ordinary gravitational mass by only a minus sign.

1. Introduction

In a previous paper [1], it was demonstrated that although the mass-energy between the plates of a Casimir cavity is negative, the corresponding inertial mass exerts a resistance to acceleration as does regular, positive matter. This is straightforward to visualize; the energy between the Casimir plates is comprised of positive zero-point radiation, which appears negative when compared with the zero-point radiation outside of the plates. The positive zero-point radiation between the plates resists acceleration, despite having a negative inertial mass-energy. On this basis, it was concluded that, generally, exotic matter exhibits inertial properties identical to those of ordinary matter [1].

The next natural question to ask is whether or not negative matter exhibits normal gravitational properties. Conventionally, exotic matter is thought to produce repelling gravitational fields; this seems to result simply when the gravitational mass carries a minus sign. But is the minus sign alone enough for us to rest assured that exotic matter acts in any way differently than ordinary matter? Considering the inertial properties of exotic matter mentioned above, it should be clear that we cannot necessarily assume anything about exotic matter. The objective of the following sections is to use classical, Newtonian gravitation to provide a clear demonstration that exotic matter does indeed exhibit repelling gravitational properties.

2. Gravitational Potential near Sphere of Exotic Matter

As mentioned above, the energy density between the plates of the Casimir cavity appears negative relative to the essentially zero energy density outside the plates. Were we to apply Newtonian gravitation to such a system, the negative energy density guarantees that a repelling gravitational field results. To avoid this problem, we shall consider an idealized system wherein a sphere of matter, *A*, having a mass-density ρ_1 is surrounded by a spherical region, *B*, having a mass-density ρ_2. As shown in Figure 1, sphere *A* has a radius *a*, and spherical region *B* has a much larger radius, $b >> a$. Both ρ_1 and ρ_2 are positive, with ρ_2 being greater than ρ_1 according to $\rho_2 = \rho_1 + \Delta\rho$. Setting ρ_1 less than ρ_2 while keeping both values positive is one way to eliminate the need for arbitrarily introducing a minus sign, while maintaining a relationship between ρ_1 and ρ_2 that is analogous to that existing with a Casimir cavity in free space.

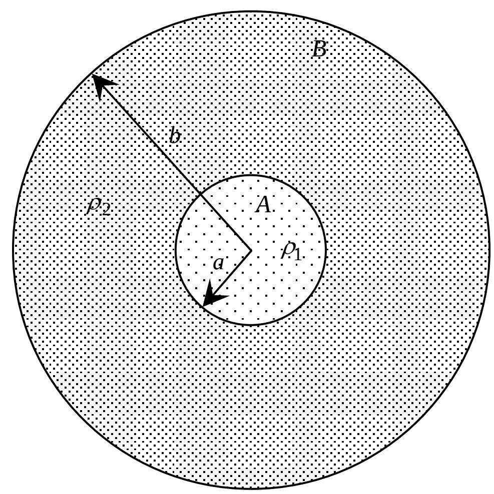

Figure 1. A sphere of matter, *A*, having a mass-density ρ_1 is surrounded by a spherical region, *B*, having a mass-density $\rho_2 > \rho_1$. Sphere *A* has a radius *a*, and spherical region *B* has a radius $b >> a$.

Of particular interest is the region outside sphere *A* at radial distances less than *b*. The gravitational potential ϕ within this region can be determined by using the familiar expression [2]

$$\phi(\mathbf{x}) = -G\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, d^3x' \tag{1}$$

where $\mathbf{x}'$ is a position vector, $\rho(\mathbf{x}')$ is the mass-density of the gravitational source, *G* is the gravitational constant, and the integration is taken over primed quantities. Also, we may use the expression [3]

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_{lm}^*(\theta',\phi') Y_{lm}(\theta,\phi) \tag{2}$$

where $r_<$ is the smaller of $\mathbf{x}$ and $\mathbf{x}'$, $r_>$ is the larger of $\mathbf{x}$ and $\mathbf{x}'$, and $Y_{lm}(\theta,\phi)$ is given by

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi}$$

wherein P_l^m are the associated Legendre functions. Substituting Eq. (2) into Eq. (1) gives

$$\phi(\mathbf{x}) = -4\pi G\int \rho(\mathbf{x}')\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi)d^3x' \quad (3)$$

For the special case of spherical symmetry, Eq. (2) simplifies to

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{r_>} \quad (4)$$

Using this and performing the angular integrations in Eq. (3) leads directly to

$$\phi(\mathbf{x}) = -4\pi G\int_{r_1}^{r_2}\frac{\rho(r')}{r_>}r'^2 dr' \quad (5)$$

For radial distances within the region $a < r < b$ Eq. (5) becomes

$$\phi(\mathbf{a} < \mathbf{x} < \mathbf{b}) = -4\pi G\left\{\frac{\rho_1}{r}\int_0^a r'^2 dr' + \frac{\rho_2}{r}\int_a^r r'^2 dr' + \rho_2\int_r^b r' dr'\right\} \quad (6)$$

Upon carrying out the integrations and noting that the volume of a sphere having a radius a is given by $V_a = 4\pi a^3/3$, we find that the gravitational potential takes the form

$$\phi(\mathbf{a} < \mathbf{x} < \mathbf{b}) = -G\frac{V_a}{r}(\rho_1 - \rho_2) - 4\pi G\rho_2\left(\frac{b^2}{2} - \frac{r^2}{6}\right) \quad (7)$$

As a further simplification, recall that ρ_1 and ρ_2 are related to one another by $\rho_2 = \rho_1 + \Delta\rho$. Substituting this relation into the first term of Eq. (7) leads to

$$\phi(\mathbf{a} < \mathbf{x} < \mathbf{b}) = G\frac{V_a}{r}\Delta\rho - 4\pi G\rho_2\left(\frac{b^2}{2} - \frac{r^2}{6}\right) \quad (8)$$

This is the gravitational potential outside sphere *A* within the region $a < r < b$.

Upon inspecting Eq. (8), it is immediately obvious that the first term in Eq. (8) provides a positive contribution which arises due to the absence of matter that would otherwise occupy the volume V_a in absence of sphere *A*. This idealized system is analogous to a Casimir cavity wherein the energy-density within the cavity is lower than outside. Viewed in this manner, Eq. (8) provides evidence that exotic matter gives rise to repelling gravitational fields.

The gravitational field outside sphere *A* is easily determined by using $\mathbf{g} = -\hat{\mathbf{r}}\partial\phi/\partial r$. Carrying this out and performing some minor algebraic manipulation leads directly to

$$\mathbf{g}(\mathbf{a} < \mathbf{x} < \mathbf{b}) = \frac{G}{r^2}\left(V_a\Delta\rho - \frac{4\pi}{3}r^3\rho_2\right)\hat{\mathbf{r}} \quad (9)$$

This expression can be simplified further upon noticing that the volume of a Gaussian sphere of radius r is $V_r = 4\pi r^3/3$. Equation (9) then becomes

$$\mathbf{g}(\mathbf{a} < \mathbf{x} < \mathbf{b}) = \frac{G}{r^2}(V_a\Delta\rho - V_r\rho_2)\hat{\mathbf{r}} \quad (10)$$

Equation (10) suggests that the difference in mass-density $\Delta\rho$ within sphere *A* gives rise to a gravitational field which opposes the field due to the Gaussian sphere. This makes sense: some matter is missing from sphere *A* and thus the first term in Eq. (10) diminishes the field that would otherwise exist were the Gaussian sphere filled entirely with mass-density ρ_2. Applying this to the case of exotic matter, however, suggests that the sphere of exotic matter provides a gravitational influence in opposition to that encountered with ordinary matter. It is straightforward to see that for the case when $\rho_2 = 0$, which implies a matter-free region for $r > a$, the gravitational field outside sphere *A* is given simply as

$$\mathbf{g}(\mathbf{a} < \mathbf{x} < \mathbf{b}) = G\frac{V_a\Delta\rho}{r^2}\hat{\mathbf{r}} \quad (11)$$

This expression clearly shows that exotic matter gives rise to gravitational fields having an orientation opposite to that due to ordinary, positive matter configurations. This result is particularly interesting in view of the fact that only positive matter was considered in deriving Eq. (10). The repelling contribution to the gravitational field arises not because exotic matter is somehow *different* than ordinary matter, but rather arises due the particular arrangement of ordinary matter within the system in question.

As a final thought, it is interesting to notice that when $\Delta\rho$ is expressed in terms of mass via

$$\Delta\rho = m'/V_a\ ,$$

where m' is the total exotic mass associated with $\Delta\rho$, the gravitational field given by Eq. (11) can written in the familiar form

$$\mathbf{g}(\mathbf{a} < \mathbf{x} < \mathbf{b}) = G\frac{m'}{r^2}\hat{\mathbf{r}} \quad (12)$$

This expression seems to suggest that we can, indeed, determine gravitational fields due to exotic matter by merely switching the sign of the gravitational source appearing in Eq. (1).

3. Force Between Matter & Exotic Matter

As a quick check of the validity of Eq. (12), we may consider an expression of the gravitational force on a stationary Casimir cavity, given in [4, 5] as

$$\mathbf{F} = \frac{\pi^2 L^2 \hbar}{720 s^3 c} g\hat{\mathbf{r}} \tag{13}$$

Equation (13) is a lift force exerted on the cavity due to its negative energy content, where L^2 is the plate area of the cavity, s is the plate separation, g is the magnitude of the gravitational field due to m_0, and $\hat{\mathbf{r}}$ is a unit vector pointing from m_0 to m'. As shown in [1], the mass due to the negative energy-density within the Casimir cavity is

$$m' = -\frac{\pi^2 L^2 \hbar}{720 a^3 c} \tag{14}$$

Using this expression and noting that the magnitude of the gravitational field is $g = -Gm_0/r^2$, Eq. (13) becomes

$$\mathbf{F} = G\frac{m' m_0}{r^2}\hat{\mathbf{r}} \tag{15}$$

Equation (15) is identical to the product of Eq. (12) and the ordinary mass m_0. Thus, the analysis presented herein appears to be in agreement with the more complex analyses presented in [4, 5].

4. Equivalence Principle

Equation (12) suggests that ordinary matter and exotic matter may perform a similar function in gravitation as positive and negative electric charges do in electromagnetism. Indeed, gravitation may behave more like electromagnetism than previously thought.

At first sight, however, there appears to be a problem with this interpretation: the Equivalence Principle states that, generally, the inertial mass ought to be equal to the gravitational mass, $m_i = m_g$. Equation (12) indicates that the active gravitational mass of exotic matter is negative. The expression of the force on a Casimir cavity derived in [4, 5] indicates that the passive gravitational mass of exotic matter is negative. As shown in [1], however, exotic matter comprises positive photonic energy which resists acceleration under the action of external forces, just as does ordinary matter. The results of [1] seem to suggest that the inertial mass of exotic matter is positive while the gravitational mass is negative, in contradiction to the Equivalence Principle.

After some thought, however, this may not appear to be such a problem, after all. If the inertial mass is negative under all circumstances, not only does the expression $\mathbf{F} = m_i\mathbf{a}$ lead to counterintuitive results, but there would be no inertial resistance to counter the force given by Eq. (13). This last point suggests that ordinary matter would repel exotic matter with an arbitrarily large acceleration. On the other hand, if the inertial mass of exotic matter resists acceleration, it provides a resistance force which counteracts the repulsion due to ordinary matter. In essence, the inertia of exotic matter ensures that bodies of exotic matter do not "feel their own weight" while in motion due to gravitation. If this is indeed the case, it would seem that violation of the Equivalence Principle is necessary in order to maintain proper and intuitive behavior of moving exotic matter. It remains to determine whether or not exotic matter follows geodesic motion similarly to ordinary matter.

5. Closing Remarks

In the preceding sections, a simple, idealized model was used to determine whether or not exotic matter produces repelling gravitational fields. As pointed out in the Introduction, exotic matter is traditionally viewed as possessing negative gravitational mass, which automatically guarantees repelling gravitational fields. In an attempt to determine whether or not such an approach is valid, Newtonian gravitation was applied to the case of a low-density spherical mass embedded within a higher density region of matter. Despite the positive nature of both matter regions, it was found that embedding the low mass-density sphere within the region of greater mass-density produces a repulsive contribution to the gravitational field within the region of greater mass-density. Upon applying this model to the case of exotic matter, such as arising due to the negative energy-density within a Casimir cavity, it was demonstrated that exotic matter does indeed produce repelling gravitational fields. Moreover, it was determined that gravitational fields due to exotic matter can be determined by considering negative gravitational mass distributions.

It is the hope of the present author that the analysis and insights presented herein will aid in the search for advanced technologies such as propellantless propulsion [6, 7].

Notes and References

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