Critique Of C. T. Ridgely's, "Applying Relativistic Electrodynamics To A Rotating Material Medium" Am. J. Phys. 66 (2) Feb. 1998

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Abstract

Ridgley's proposed 'most simple' generalization of the Minkowski constitutive relations is shown to be invalid as being inconsistent with the definitions of the proper quantities of electric permittivity and magnetic permeability as defined in the rest frame of a material, and more importantly, Ridgley's proposed generalization is inconsistent with the fundamental field equations when the material "sources" are included.

In his article, "Applying Relativistic Electrodynamics To A Rotating Material Medium", Ridgley proposed a 'most simple' generalization of the Minkowski constitutive equations for use in rotating systems. As a consequence of this generalization Ridgley obtains the constitutive equations in the magnet rest frame that are given by eqs. (23a) and (23b). While eq. (23a) is the standard definition for the electric permittivity ε in a material's rest frame, eq. (23b) is not the standard definition for the magnetic permeability μ in a material's rest frame. According to eq. (23b) even with μ equal to one there will nevertheless be a non-zero magnetization dependent upon ε . This dependence is inconsistent with ε and μ being defined as independent scalar invariants representing *separately* the material's electric polarization and magnetization properties respectfully.

More importantly, Ridgley's proposed field equations [eqs. (18)] are in contradiction to the fundamental field equations as correctly expressed in eq. (4) of his paper. To demonstrate this we first take eq. (18a) (i.e., the field equation in the rotating frame, denoted by primed variables, with only the free charge density ρ' as a source) and add the material charge density ρ'_b as sources thus obtaining the field equation for the total charge density as a source as follows:

$$\nabla \cdot (\varepsilon \vec{E}' - \vec{v} \times \frac{1}{\mu} \vec{B}' - 4\pi \vec{P}') = 4\pi(\rho' + \rho_b') \qquad (1)$$

where we have used the following relation for the material electric dipole moment density generally valid in any coordinate system:

$$\nabla \cdot \vec{P}' = -\rho_b'$$

Substituting Ridgley's constitutive equation eq. (23a) for the electric dipole moment density into eq. 1 it then gives

$$\nabla \cdot (\varepsilon \vec{E}' - \vec{v} \times \frac{1}{\mu} \vec{B}' - [\varepsilon - 1]\vec{E}') = 4\pi(\rho' + \rho_b')$$

$$\nabla \cdot (\vec{E}' - \vec{v} \times \frac{1}{\mu} \vec{B}') = 4\pi(\rho' + \rho_b') \qquad (2)$$

Likewise we take eq. (18d) (i.e., the field equation with only the free current density $\vec{j}'$ as a source) and add the material current density $\vec{j}_b'$ as sources thus obtaining the field equation for the total current density as a source as follows:

$$\nabla \times (\frac{1}{\mu}[\overline{B} - \vec{v} \times \vec{E}'] + 4\pi \vec{M}) = 4\pi(\vec{j}' + \vec{j}_b') \qquad (3)$$

where we have used the following relation for the material magnetic dipole moment density also generally valid in any coordinate system (assuming units that velocity of light is one, and no time dependent fields):

$$\nabla \times \vec{M} = \vec{j}_b'$$

Substituting Ridgley's constitutive equation eq. (23b) for the magnetic dipole moment density (neglecting always second order terms) into eq. 3 it then gives:

$$\nabla \times \left(\frac{1}{\mu}[\overline{B}' - \vec{v} \times E'] + (1 - \frac{1}{\mu})\vec{B}' + (\frac{1}{\mu} - \varepsilon)\vec{v} \times \vec{E}'\right) = 4\pi(\vec{j}' + \vec{j}_b')$$

$$\nabla \times (\overline{B}' - \varepsilon\vec{v} \times E') = 4\pi(\vec{j}' + \vec{j}_b') \qquad (4)$$

However, eq. 2, which contains μ , and eq. 4, which contains ε , are clearly not correct since the field equations for the *total charge and current densities* (total vacuum sources) must be *independent* of the material's electric permittivity ε and the magnetic permeability μ as can also be seen from the application of the fundamental field equations (applying eq. (4) in Ridgley's article and again neglecting second order terms) which gives:

$\nabla \cdot (\vec{E}' - \vec{v} \times \vec{B}') = 4\pi(\rho' + \rho_b')$ and

$$\nabla \times (\overline{B}' - \vec{v} \times E') = 4\pi(\vec{j}' + \vec{j}_b')$$