On the Origin of Inertia

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In a previous analysis, it was argued that greater insight into the origin of inertia cannot be obtained solely on the basis of inertial mass; the participation of space-time must also be taken into account. Special and general relativity were used to show that the origin of inertia is the distortion of time in accelerating systems of reference. Herein, the behavior of light in accelerating coordinates is used to provide further evidence that time distortion is the source of inertia. We consider a sealed vessel that contains a photon gas and resides in a uniform acceleration field. It is shown that radiation pressure exerts a net force on the vessel whenever space-time structure gives rise to a relativistic Doppler-shift that is observable in the co-moving reference frame of the vessel. Then the force is expressed in terms of space-time distortion in the reference frame of an infinitesimal small vessel residing in a Newtonian acceleration field. Carrying this out leads to an expression in agreement with the previous analysis, in which the force of inertia is derived on the basis of special and general relativity. The resulting expression implies that any entity possessing energy, regardless of embodiment, exhibits inertial properties; and more particularly, that the origin of inertia is the relativistic nature of time.

1. Introduction

There can be no doubt that the longest standing mystery in the field of physics is the means by which ordinary matter resists accelerated motion (inertia). A seemingly straightforward approach to the problem of inertia is to express the inertial mass m appearing in Newton's second law of motion,

$$\mathbf{F} = \frac{d}{dt}\left(m\mathbf{v}\right) \tag{1}$$

in terms of some other entity or interaction. The usual problem with such an approach, however, is that it ascribes the force of inertia entirely to inertial mass itself, while completely ignoring any contribution made by space-time. According to general relativity, inertial and gravitational forces alike arise directly out of the structure of space-time [1-6]. Thus, it seems reasonable that the problem of inertia must be formulated on the basis of both matter and space-time structure.

In a previous analysis [7], special and general relativity were used to show that inertia arises out of the intimate relationship between the energy content of matter and the local structure of space-time. More specifically, the analysis showed that the inertial resistance force of a body undergoing uniform acceleration in flat, Minkowski space-time can be expressed as

$$\mathbf{F} = -E_0\bar{\nabla}\left(\frac{dt}{d\tau}\right) \tag{2}$$

where E_0 is the rest-mass energy of the body, and $dt/d\tau$ expresses the distortion of time in the co-moving reference frame (CMRF) of the body relative to Minkowski space-time [8]. It was pointed out that, as a given body accelerates under the influence of an external force, proper time in the CMRF of the body becomes increasingly distorted relative to time experienced by observers in Minkowski space-time. The changing time distortion gives rise to a second force on the body that has magnitude equal to, direction opposite to, the externally applied force. Assuming no other forces act on the body, the second force is the inertial resistance of the body, measurable in Minkowski space-time. The form of the force, given by Eq. (2), led to the conclusion that the origin of inertia is relativistic time distortion [7-8].

In the present analysis, the behavior of light in accelerating coordinates is used to provide further evidence that inertia has its origin in time distortion. The motivation for this approach is simple. As pointed out above, an adequate description of inertia must take into consideration both matter and space-time structure. When considering a material body undergoing accelerated motion, it is easy to lose sight of the relationship between the body and the space-time continuum in which the body moves. One is then led down the path of ascribing inertia entirely to the inertial mass m appearing in Eq. (1). Light, on the other hand, seems to be free of this difficulty. The bending of light beams in gravitational fields [9] makes it quite clear that the behavior of photons cannot be considered without taking the behavior of space-time into account. In addition, photons each have energy $E = h\nu$, where h is the Planck constant and ν is the frequency of radiation. Thus, by virtue of their energy content, we expect that photons must exhibit inertial properties [10-12]. And, as with ordinary bulk matter, we expect that the origin of those properties must be the relativistic distortion of time that arises out of the local structure of space-time [7-8].

2. Radiation Pressure in an Accelerating Reference Frame

As mentioned in the preceding Section, since photons each have energy, we expect that light should have inertia in accordance with Eq. (2). To demonstrate this, consider the ideal vessel depicted by Fig. 1. Mounted on wall A is a photon source S_A that produces monochromatic photons of energy $E_0 = h\nu_0$, where ν_0 is the frequency according to observers residing at wall A. The vessel is sealed, and we assume the interior surfaces are nearly perfectly reflective. Thus, photons are prohibited from escaping into free space, and the dissipation of energy to the walls of the vessel is minimized. As a further precaution against photon loss [13], assume that photon source S_A emits photons in such a manner that an average number of photons $\overline{N}$ is maintained in the vessel at all times.

It is straightforward to show that when such a vessel resides in an inertial system, as shown in Fig. 1, the total force imparted to the vessel due to radiation pressure is zero. Radiation pressure on one wall annuls that on the opposing wall. However, when the vessel resides in an acceleration field, as depicted by Fig. 2, observers in the CMRF of the vessel, S', find that the radiation pressure on walls A and B is not equivalent. This difference in radiation pressure leads to an observable force on the vessel [14].

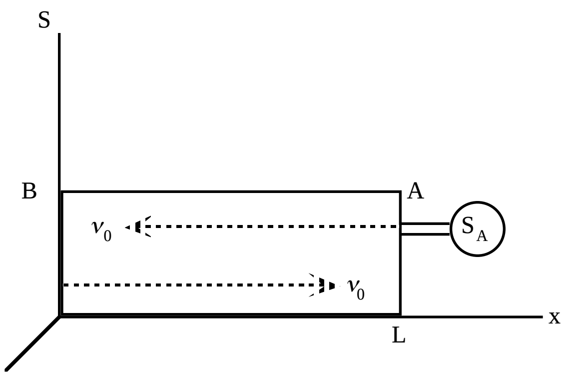

Figure 1. A sealed vessel of length L contains a photon gas and resides in an inertial system of reference S. At wall A is a photon source S_A that produces a steady stream of monochromatic photons of frequency ν_0. To minimize photon loss, the interior surfaces of the vessel are nearly perfectly reflective and photon source S_A maintains an average number of photons $\overline{N}$ in the vessel at all times.

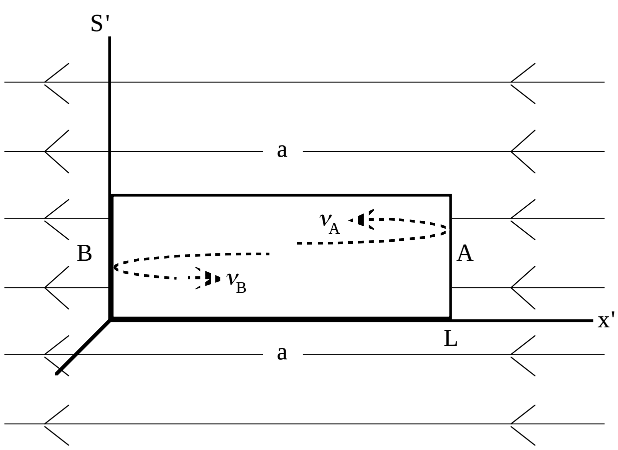

Figure 2. A vessel of length L, containing a photon gas, resides in a uniform acceleration field oriented in the $-x'$ direction of system S'. The frequency of radiation observed at walls A and B is ν_A and ν_B, respectively.

To derive the force on the vessel, we first find the total momentum transferred to the vessel. The momentum transferred to wall A due to the reflection of $\overline{N}$ photons is [15]

$$\Delta\overline{\mathbf{p}}_A = \frac{\overline{N}h\nu_A}{c}\hat{\mathbf{i}} - \left(-\frac{\overline{N}h\nu_A}{c}\hat{\mathbf{i}}\right) = \frac{2\overline{N}h\nu_A}{c}\hat{\mathbf{i}} \tag{3}$$

where $\hat{\mathbf{i}}$ is a unit vector in the x' coordinate direction and ν_A is the frequency observed at wall A. A similar calculation gives the momentum transferred to wall B:

$$\Delta\overline{\mathbf{p}}_B = -\frac{2\overline{N}h\nu_B}{c}\hat{\mathbf{i}} \tag{4}$$

Using Eqs. (3) and (4), the total momentum imparted to the vessel by radiation pressure is then

$$\Delta\overline{\mathbf{p}} = \Delta\overline{\mathbf{p}}_A + \Delta\overline{\mathbf{p}}_B = \frac{2\overline{N}h}{c}\left(\nu_A - \nu_B\right)\hat{\mathbf{i}} \tag{5}$$

With Eq. (5) in hand, the force exerted on the vessel can be obtained by using $\mathbf{F} = \Delta\mathbf{p}/\Delta\tau$, where $\Delta\tau$ is an interval of proper time in the CMRF of the vessel. Note that the photons' change in momentum takes place during the time in which they traverse a round trip path across the length of the vessel. And, the vessel's momentum changes during the time in which $\overline{N}$ photons complete such a trip. Thus we use $\Delta\tau = 2L/c$, where L is the length of the vessel. The force is then

$$\overline{\mathbf{F}} = \overline{N}h\left(\frac{\nu_A - \nu_B}{L}\right)\hat{\mathbf{i}} \tag{6}$$

This is the proper force that radiation pressure exerts on the vessel. The force is non-zero in any instance in which a relativistic Doppler-shift, $\nu_A - \nu_B \neq 0$, is observable in the CMRF of the vessel. Observers residing on the vessel can determine the magnitude of the force by simply measuring the frequency of radiation received at walls A and B.

3. Relativistic Time Distortion as the Source of Inertia

The previous Section showed that radiation pressure exerts a force on a vessel whenever a relativistic Doppler-shift arises in the CMRF of the vessel. An important point to notice is that such a shift in frequency is a direct manifestation of space-time structure within the vessel. This being the case, Eq. (6) implies that the force on the vessel arises directly out of the local structure of space-time. The dependence on space-time can be made more apparent by expressing Eq. (6) in terms of the frequency-shift that occurs between the walls of a uniformly accelerating vessel.

With photon source S_A mounted on wall A, observers at that wall measure the frequency of light as it is initially emitted into the vessel. Thus, the frequency observed at wall A is simply the proper frequency

$$\nu_A = \nu_0 \tag{7}$$

As the photons travel from wall A to wall B, in the direction of the acceleration field, they gain energy. As a result, photons arriving at wall B are blue-shifted. According to observers at wall B, the frequency of radiation is

$$\nu_B = \nu_0 \sqrt{\frac{1 + V_R/c}{1 - V_R/c}} \tag{8}$$

where V_R is the relative velocity gained by wall B during the time in which photons traverse the vessel from wall A to wall B. After detection at wall B, photons are then reflected back toward wall A. As the photons traverse the vessel in opposition to the acceleration field, they expend energy. Assuming that the acceleration of the vessel is constant, conservation of energy implies that the energy expended in traveling from wall B to wall A is equivalent to the energy gained in traveling from wall A to wall B. Consequently, when the photons return to wall A, observers at wall A find that the frequency of radiation is again the proper frequency given by Eq. (7).

Upon substituting Eqs. (7) and (8) into Eq. (6), the force on the vessel becomes

$$\bar{\mathbf{F}} = \frac{\bar{N} h\nu_0}{\Delta x'} \left(1 - \sqrt{\frac{1 + V_R/c}{1 - V_R/c}} \right) \hat{\mathbf{i}} \tag{9}$$

where $\Delta x'$ is the vessel's length along the x' coordinate axis. This expression can be simplified by noting that proper time at wall B is dilated relative to proper time at wall A according to [16]

$$\frac{d\tau_A}{d\tau_B} = \frac{1}{\sqrt{1 - V_R^2/c^2}} \tag{10}$$

where $d\tau_A$ and $d\tau_B$ are intervals of proper time at walls A and B, respectively. After some algebraic fiddling, Eq. (9) can then be expressed in the form

$$\bar{\mathbf{F}} = \frac{\bar{N} h\nu_0}{\Delta x'} \left(1 - \frac{d\tau_A}{d\tau_B} - \frac{V_R}{c} \frac{d\tau_A}{d\tau_B} \right) \hat{\mathbf{i}} \tag{11}$$

Next, note that the force can be written in terms of coordinate time in the accelerating system as

$$\bar{\mathbf{F}} = \frac{\bar{N} h\nu_0}{\Delta x'} \left\{ 1 - \frac{dt}{d\tau_B} \frac{d\tau_A}{dt} \right\} \hat{\mathbf{i}} - \frac{\bar{N} h\nu_0}{c} \frac{V_R}{\Delta x'} \frac{d\tau_A}{d\tau_B} \hat{\mathbf{i}} \tag{12}$$

And upon factoring $(d\tau_A/dt)$ out of the numerator of the first term, the force becomes

$$\bar{\mathbf{F}} = \frac{\bar{N} h\nu_0}{\Delta x'} \left[\frac{d\tau_A}{dt} \left(\frac{dt}{d\tau_A} - \frac{dt}{d\tau_B} \right) - \frac{V_R}{c} \frac{d\tau_A}{d\tau_B} \right] \hat{\mathbf{i}} \tag{13}$$

This expression implies that radiation pressure exerts a net proper force on the vessel whenever a distortion [8] of space-time exists between the walls of the vessel.

The connection between Eqs. (13) and (2) can be made by considering a vessel in the point-particle limit; that is, a vessel of infinitesimal dimensions. We do this by taking the limit of Eq. (13) as $d\tau_B$ tends toward $d\tau_A$ and $\Delta x'$ tends toward zero. Carrying this out gives

$$\begin{aligned} \bar{F}_x(x) &= \bar{N} h\nu_0 \lim_{\substack{\Delta x' \to 0 \\ d\tau_B \to d\tau_A}} \frac{1}{\Delta x'} \left(\frac{d\tau_A}{dt} \left(\frac{dt}{d\tau_A} - \frac{dt}{d\tau_B} \right) - \frac{d\tau_A}{d\tau_B} \frac{V_R}{c} \right) \\ &= -\bar{N} h\nu_0 \frac{d\tau}{dt} \frac{\partial}{\partial x'} \left(\frac{dt}{d\tau} \right) \end{aligned} \tag{14}$$

where the minus sign in the first term arises because $(dt/d\tau_B) > (dt/d\tau_A)$, and the second term goes to zero because $V_R << c$ for small vessels. Equation (14) can then be written in three-dimensional notation as

$$\bar{\mathbf{F}} = -\bar{N} h\nu_0 \bar{\nabla}' \left\{ \ln\left(\frac{dt}{d\tau} \right) \right\} \tag{15}$$

This is the inertial resistance of a particle of energy $\bar{E}_0 = \bar{N} h\nu_0$, according to observers residing in the CMRF of the particle. Equation (2) can now be obtained either by transforming Eq. (15) from the accelerating frame of the particle to an inertial frame or by considering the case of a weak, Newtonian acceleration field. Choosing the latter approach, we note that for the case of Newtonian acceleration, we can take $\ln(dt/d\tau) \cong (dt/d\tau) - 1$. Using this relation in Eq. (15) and noting that the energy content of the particle is $\bar{E}_0 = \bar{N} h\nu_0$ leads to an expression identical to Eq. (2):

$$\bar{\mathbf{F}} = -\bar{E}_0 \bar{\nabla}' \left(\frac{dt}{d\tau} \right) \tag{16}$$

This is the inertial resistance of a particle of energy $\bar{E}_0$ that resides in a region of space-time in which time is distorted according to $dt/d\tau$. From this result we can deduce that all forms of energy, regardless of embodiment, resist changes to their states of motion, and that the source of such resistance is relativistic time distortion arising due to acceleration [7-8].

4. Discussion

As mentioned in the Introduction, one approach to the problem of inertia is to express the inertial mass in Newton's second law of motion in terms of some other more fundamental entity or interaction. A previous analysis argued that such an approach cannot yield any deeper insight into the origin of inertia. The problem of inertia must take into account both matter and the structure of space-time. On that basis, special and general relativity were used to show that the origin of the inertial properties of matter is the distortion of time arising in accelerating systems of reference [7].

The present analysis has attempted to provide further evidence that time distortion is the source of inertia. It began by considering a sealed vessel that contains a photon gas. When such a vessel resides in an inertial reference frame, radiation pressure is uniformly distributed on the interior surfaces of the vessel. Thus, the total force imparted to the vessel is zero. It was pointed out, however, that this is no longer the case when the vessel resides in a distorted region of space-time, such as due either to uniform acceleration or gravitation. When an acceleration field is present, observers at rest on the vessel find that the frequency of light traversing the vessel becomes Doppler-shifted. Consequently, the observers find that radiation pressure is no longer uniformly distributed within the vessel [14], giving rise to a net proper force on the vessel.

The crucial point to be noticed is that the Doppler-shift arises directly out of the structure of space-time. Therefore, the force acting on the vessel is a direct manifestation of the structure of space-time within the vessel. To make the role played by space-time more apparent, we considered the case of a uniformly accelerating vessel. Carrying this out, we expressed the force on the vessel in terms of the Doppler-shift arising within the vessel. We found that radiation pressure exerts a net force on the vessel whenever space-time is distorted [8] within the dimensions of the vessel.

We then considered the case of an infinitesimally small vessel undergoing uniform Newtonian acceleration. This led to an expression for the force in agreement with previous analyses, employing special and general relativity [7]. The form of the force expression leads us to conclude that any entity possessing energy resists changes to its state of motion, and that the source of such resistance is the local structure of time in accelerating systems of reference.

References

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[8] We use the word "distortion" in reference to any instance in which local space-time structure deviates from the flat, Minkowski space-time of special relativity.

[9] See, for example, H. Ohanian and R. Ruffini, Ref. [5], pp. 179-193; and I. R. Kenyon, Ref. [6], pp. 15-20.

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[13] For a discussion on the inherent difficulties with maintaining a photon gas, see G. Cook and R. H. Dickerson, "Understanding the chemical potential," Am. J. Phys. **63**, 737-742.

[14] A similar model was used by A. Einstein to show that $E = mc^2$. See, for example, R. Resnick, **Introduction to Special Relativity** (Wiley, New York, 1968), pp. 140-141.

[15] Here, we note that momentum components that are parallel to the walls of the vessel sum to zero. Thus, we consider only momentum components that are perpendicular to walls A and B.

[16] A similar example in which proper time is related between observers residing in a gravitational field can be found in H. Ohanian and R. Ruffini, Ref. [5], pp. 167-168.